* Computation of N point DFTs of two real sequences by performing DFT computation (matrix computation) only once.

$x_1(n)$ & $x_2(n)$ → 2 real sequences of length N each

$X_1(k)$ & $X_2(k)$ → their respective N point DFTs.

These N point DFTs can be computed efficiently using a single N point DFT $X(k)$ of a complex length N sequence given by.

$$x(n) = x_1(n) + j\,x_2(n) \quad \text{—— (A)}$$

where $x_1(n) = \text{Re}\{x(n)\}$

& $x_2(n) = \text{Im}\{x(n)\}$

Now $x^*(n) = x_1(n) - j\,x_2(n)$ —— (B)

Adding A & B

$$x(n) + x^*(n) = 2x_1(n)$$

$$\therefore x_1(n) = \frac{x(n) + x^*(n)}{2}$$

For a complex sequence

$$x^*(n) \xleftrightarrow[N]{DFT} X^*(-k) = X^*(N-k) \quad \text{conjugate symmetry}$$

Taking DFT

$$\boxed{X_1(k) = \frac{X(k) + X^*(-k)}{2}}$$

Subtracting (B) from (A) $x(n) - x^*(n) = 2j\,x_2(n)$

$$\therefore x_2(n) = \frac{x(n) - x^*(n)}{2j}$$

Taking DFT

$$\boxed{X_2(k) = \frac{X(k) - X^*(-k)}{2j}}$$

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Ex. 1) ⓐ $x(n) = \{1+5j, 2+6j, 3+7j, 4+8j\}$ Find DFT $X(k)$

ⓑ Using results obtained in ⓐ & not otherwise find DFT of following sequences.

i) $x_1(n) = \{1, 2, 3, 4\}$

ii) $x_2(n) = \{5, 6, 7, 8\}$

a) $X = Wx$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+5j \\ 2+6j \\ 3+7j \\ 4+8j \end{bmatrix} = \begin{bmatrix} 10+26j \\ -4 \\ -2-2j \\ -4j \end{bmatrix}$$

$$X(k) = \{10+26j, -4, -2-2j, -4j\}$$

b) $x(n) = x_1(n) + j x_2(n)$

$x^*(n) = x_1(n) - j x_2(n)$

$x(n) + x^*(n) = 2x_1(n)$

$$x_1(n) = \frac{x(n) + x^*(n)}{2}$$

Taking DFT

$$X_1(k) = \frac{X(k) + X^*(-k)}{2}$$

$X(k) = \{10+j26, -4, -2-2j, -4j\}$

$X^*(k) = \{10-26j, -4, -2+2j, 4j\}$

$X^*(-k) = \{10-26j, 4j, -2+2j, -4\}$

$\therefore X_1(k) = \{10, -2+2j, -2, -2-2j\}$

Also $x(n) - x^*(n) = 2j x_2(n)$

$$x_2(n) = \frac{x(n) - x^*(n)}{2j}$$

Taking DFT $\quad X_2(k) = \dfrac{X(k) - X^*(-k)}{2j}$

$X_2(k) = \{26, -2+2j, -2, -2-2j\}$

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2) Two sequences are given below

$x_1(n) = \{1+5j, 2+6j, 3+7j, 4+8j\}$ &

$x_2(n) = \{6, 8, 10, 12\}$ Find DFT of both sequences using DFT only onces.

Sol^n

$X_1(k) = \{10+26j, -4, -2-2j, -4j\}$

let

$x_1(n) = p(n) + j\,q(n)$

where $p(n) = \{1, 2, 3, 4\}$

$q(n) = \{5, 6, 7, 8\}$

$P(k) = \dfrac{X(k) + X^*(-k)}{2} = \{10, -2+2j, -2, -2-2j\}$

$Q(k) = \dfrac{X(k) + X^*(-k)}{2j} = \{26, -2+2j, -2, -2-2j\}$

we find that

$x_2(n) = p(n) + q(n)$

$X_2(k) = P(k) + Q(k)$

$= \{36, -4+4j, -4, -4-4j\}$

3) Zero interpolation in one domain leads to replication in the other $x(n) = \{a, b, c, d\}$ & $X(k) = \{A, B, C, D\}$

Using above result & not otherwise find DFT of

i) $x_1(n) = \{a, 0, b, 0, c, 0, d, 0\}$

ii) $x_2(n) = \{a, 0, 0, b, 0, 0, c, 0, 0, d, 0, 0\}$

Sol^n.

i) $x_1(n) = \{a, 0, b, 0, c, 0, d, 0\}$

$$X_1(k) = \sum_{n=0}^{7} x_1(n) W_8^{nk}$$

$$= \sum_{n=0}^{3} x_1(2n) W_8^{2nk} + \sum_{n=0}^{3} x_1(2n+1) W_8^{(2n+1)k}$$

$$= \sum_{n=0}^{3} x_1(2n) W_4^{nk} + W_8^{k} \sum_{n=0}^{3} x_1(2n+1) W_4^{nk}$$

$x_1(2n) = \{a, b, c, d\}$ & $x_2(2n+1) = \{0, 0, 0, 0\}$

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We find that $x_1(2n) = x(n)$

$$\therefore X_1(k) = \sum_{n=0}^{3} x(n)\, W_4^{nk} = X(k)$$

$x_1(n)$ is an 8 point sequence

$$\therefore X_1(k) = \{A, B, C, D, A, B, C, D\}.$$

ii) $x_2(n) = \{a, 0, 0, b, 0, 0, c, 0, 0, d, 0, 0\}$

$$X_2(k) = \sum_{n=0}^{11} x_2(n)\, W_{12}^{nk}$$

$$= \sum_{n=0}^{3} x_2(3n)\, W_{12}^{3nk} + \sum_{n=0}^{3} x_2(3n+1)\, W_{12}^{(3n+1)k} + \sum_{n=0}^{3} x_2(3n+2)\, W_{12}^{(3n+2)k}$$

$$= \sum_{n=0}^{3} x_2(3n)\, W_4^{nk} + W_{12}^{k} \sum_{n=0}^{3} x_2(3n+1)\, W_4^{nk} + W_{12}^{2k} \sum_{n=0}^{3} x(3n+2)\, W_4^{nk}$$

$x_2(3n) = \{a, b, c, d\} = x(n)$

$x_2(3n+1) = \{0, 0, 0, 0\}$

$x_2(3n+2) = \{0, 0, 0, 0\}$

$$X_2(k) = \sum_{n=0}^{3} x(n)\, W_4^{nk} = X(k)$$

$$= \{A, B, C, D, A, B, C, D, A, B, C, D\}$$

We know that

$x_1(n) = x\left(\frac{n}{2}\right)$ i.e one zero betn

& $x_2(n) = x\left(\frac{n}{3}\right)$ i.e two zeros betn.

If $x\left(\frac{n}{L}\right)$ ie L-1 zeros in betn this process is known as upsampling & denoted by $x(n) \longrightarrow \boxed{L\uparrow} \longrightarrow x\left(\frac{n}{L}\right)$

Therefore whenever we upsample by L we find that we get L-1 images of the DFT of the original sequence

$\therefore x(n) = \{a, b, c, d\} \Rightarrow X(k) = \{A, B, C, D\}$

$x_1(n) = \{a, 0, 0 \ldots L-1, b, 0, 0 \ldots L-1, c, 0, 0 \ldots L-1, d, 0, 0 \ldots L-1\}$

$X_1(k) = \{A, B, C, D, A, B, C, D, \ldots$ L-1 times$\}$

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∴ Zero padding a sequence in betn will give a DFT whose values are same as the DFT of the original sequence.

$$x(n) = \{x_0, x_1, x_2, x_3\}$$

$$X(k) = \{X_0, X_1, X_2, X_3\}$$

1) $$x_1(n) = \begin{cases} x(\frac{n}{2}) & ; n \text{ even} \\ 0 & ; n \text{ odd} \end{cases}$$

$$x_1(n) = \{x_0, 0, x_1, 0, x_2, 0, x_3, 0\}$$

$$X_1(k) = \{X_0, X_1, X_2, X_3, X_1, X_2, X_3, X_4\} = \{X(k), X(k)\}$$

2) $$x_2(n) = x(\frac{n}{3})$$

$$x_2(n) = \{x_0, 0, 0, x_1, 0, 0, x_2, 0, 0, x_3, 0, 0, x_4, 0, 0\}$$

$$X_2(k) = \{X_0, X_1, X_2, X_3, X_0, X_1, X_2, X_3, X_0, X_1, X_2, X_3\}$$
$$= \{X(k), X(k), X(k)\}$$

3) $$x_L(n) = x(\frac{n}{L})$$

$$x_L(n) = \{x_0, 0, 0 --- L-1, x_1, 0, 0 --- L-1, x_2, 0, 0 -- L-1, x_3, 0, 0 --- L-1 \text{ times}\}$$

$$X_L(k) = \{X_0, X_1, X_2, X_3, X_0, X_1, X_2, X_3 ---- \}$$

$$= \underbrace{\{X(k), X(k), --- X(k)\}}_{}$$

L-fold replication.

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4) i) $x(n) = \{1, 2, 3, 4\}$ Find DFT $X(k)$

ii) Using result obtained in (i) & not otherwise find DFT of the following sequences

$x_1(n) = \{1, 0, 2, 0, 3, 0, 4, 0\}$ ie $x_1(n) = x(\frac{n}{2})$

$x_2(n) = \{1, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$ ie $x_2(n) = x(\frac{n}{3})$

ii soln.

i) $X(k) = \{10, -2+2j, -2, -2-2j\}$

$X_1(k) = \{10, -2+2j, -2, -2-2j, 10, -2+2j, -2, -2-2j\}$

$X_2(k) = \{10, -2+2j, -2, -2-2j, 10, -2+2j, -2, -2-2j, 10, -2+2j, -2, -2-2j\}$

5) Zero padding at the end of sequence.

$x(n) = \{a, b, c, d\}$ & $X(k) = \{A, B, C, D\}$

Using above result & not otherwise find DFT of

$x_3(n) = \{a, b, c, d, 0, 0, 0, 0\}$

soln For even values of k

$$X_3(k) = \sum_{n=0}^{7} x_3(n) \cdot W_8^{nk}$$

$$\therefore X_3(2k) = \sum_{n=0}^{7} x_3(n) W_4^{nk}$$

$$= \sum_{n=0}^{3} x_3(n) W_4^{nk} + \sum_{n=4}^{7} x_3(n) W_4^{nk}$$

$$x_3(n) = \begin{cases} x(n) & 0 \le n \le 3 \\ 0 & 4 \le n \le 7 \end{cases}$$

$$X_3(2k) = \sum_{n=0}^{3} x(n) W_4^{nk}$$

$$\therefore X_3(2k) = X(k)$$

$X_3(0) = A$, $X_3(2) = B$, $X_3(4) = C$, $X_3(6) = D$

$\therefore$ This indicates that zero padding at the end makes the even pt DFTs same as the original DFT written in ascending order. However odd pt DFT will be different.

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$x(n) = \{x_0, x_1, x_2, x_3\}$

$X(k) = \{X_0, X_1, X_2, X_3\}$

$x_1(n) = \{x_0, x_1, x_2, x_3, 0, 0, 0, 0\}$

$\cancel{X_1(2k) = X(k)}$

$X_1(k) = \{X(0), N.F, X(1), N.F, X(2), N.F, X(3), N.F\}$

5) i) $x(n) = \{1, 2, 3, 4\}$ Find DFT $X(k)$

ii) Using result obtained in i) & not otherwise find DFT $X_1(k)$ for even values of k & $X_2(k)$

$x_1(n) = \{1, 2, 3, 4, 0, 0, 0, 0\}$

$x_2(n) = \{1, 2, 3, 4, 1, 2, 3, 4\}$

soln i)

$X(k) = \{10, -2+2j, -2, -2-2j\}$

ii) $x_1(n) = \{1, 2, 3, 4, 0, 0, 0, 0\}$

$X_1(k) = \{10, NF, -2+2j, NF, -2, N.F, -2-2j, NF\}$

$x_2(n) = \{1, 2, 3, 4, 1, 2, 3, 4\}$

$= x_1(n) + x_1\left(n - \frac{N}{2}\right) \quad ; N = 8$

$X_2(k) = X_1(k) + (-1)^k X_1(k)$

$= 2X_1(k) \quad$ k even

$= 0 \quad$ k odd

$X_2(k) = \{20, 0, -4+4j, 0, -4, 0, -4-4j, 0\}$

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* Symmetry Properties of DFT

Let $x(n)$ be a N pt complex valued sequence

$$x(n) = l(n) + j\,m(n)$$

it's DFT $X(k) = L(k) + j\,M(k)$

By Defn

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} = \sum_{n=0}^{N-1} \left[l(n) + j\,m(n)\right] e^{-j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} \left[l(n) + j\,m(n)\right] \left[\cos\frac{2\pi nk}{N} - j\sin\frac{2\pi nk}{N}\right]$$

$$= \sum_{n=0}^{N-1} l(n)\cos\frac{2\pi nk}{N} - j\,l(n)\sin\frac{2\pi nk}{N} + j\,m(n)\cos\frac{2\pi nk}{N} + m(n)\sin\frac{2\pi nk}{N}$$

$$= \sum_{n=0}^{N-1} \left\{ \left[l(n)\cos\frac{2\pi nk}{N} + m(n)\sin\frac{2\pi nk}{N}\right] + j\left[m(n)\cos\frac{2\pi nk}{N} - l(n)\sin\frac{2\pi nk}{N}\right] \right\}$$

$$= \sum_{n=0}^{N-1} \left[l(n)\cos\frac{2\pi nk}{N} + m(n)\sin\frac{2\pi nk}{N}\right] + j\sum_{n=0}^{N-1}\left[m(n)\cos\frac{2\pi nk}{N} - l(n)\sin\frac{2\pi nk}{N}\right]$$

Real part of $X(k)$, $L(k) = \sum_{n=0}^{N-1} l(n)\cos\frac{2\pi nk}{N} + m(n)\sin\frac{2\pi nk}{N}$

Imag part of $X(k)$, $M(k) = \sum_{n=0}^{N-1} m(n)\cos\frac{2\pi nk}{N} - l(n)\sin\frac{2\pi nk}{N}$

Sequence	$x(n)$		$X(k)$		DFT
	$l(n)$ Real part	$m(n)$ Imag part	$L(k)$ Real Part	$M(k)$ Imag	
Real & Even	even	0	even	0	Real & even
Real & odd	odd	0	0	odd	imaginary & odd
Imag & even	0	even	0	even	Imaginary & even
Imag & odd	0	odd	odd	0	real & odd.

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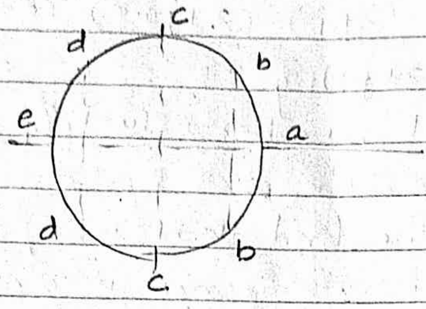
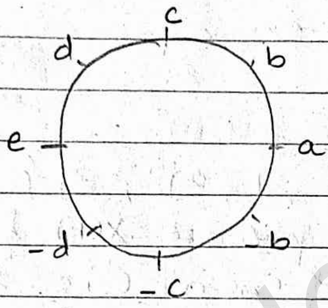
Consider $x(n) = \{a, b, c, d, e, d, c, b\}$ a, b, c, d, e are real

we Find that $x(n) = x(-n)$. This sequence is real & even



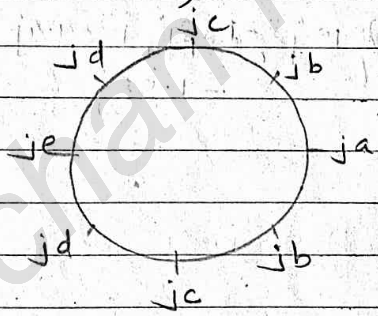
Consider $x(n) = \{a, b, c, d, e, -d, -c, -b\}$

we find that $x(n) = -x(-n)$ This sequence is real & odd



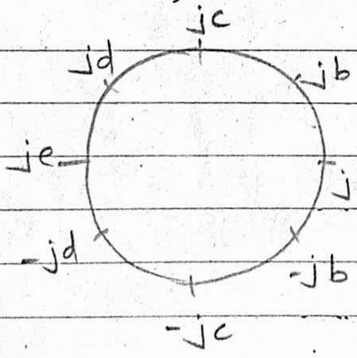
Consider $x(n) = \{ja, jb, jc, jd, je, jd, jc, jb\}$

we find $x(n) = x(-n)$. This sequence is imaginary & even.



Consider $x(n) = \{ja, jb, jc, jd, je, -jd, -jc, -jb\}$

we find that $x(n) = -x(-n)$. This sequence is imaginary & odd.



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Ex. 1) $x(n) = \{a, b, c, 1, 0, 1, c, b\}$ can this sequence have $X(k)$ as shown below.

$X(k) = \{(2+a+2b+2c), (3+j), (2+j0.5), (-3+2j), 2, (-3+j2), (2+j0.5), (3+j4)\}$

where a, b, c are real numbers. Justify your Ans.

Sol^n –

From sequence, we find that

$x(n) = x(-n)$ i.e sequence is even & also real

The DFT of a real & even sequence will always be real & even ∴ This sequence cannot have the DFT $X(k)$ that is given.

2) If $x(n) \longleftrightarrow X(k)$ &

$x(n) = \{j, -2j, j, 4j, j, 2j, -2j\}$

Then complete the sequence $X(k) = \{15j, -1.58j, j, -4.4j, 3, \_\}$

Sol^n $x(n)$ is imaginary & even (ie $x(n) = x(-n)$)

An imag & even sequence will always have a DFT which is also img & seven

∴ $X(k) = X(-k)$ & $X(k) = \{15j, -1.58j, j, -4.4j, 3j, -4.4j, j, -1.58j\}$

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PARSEVAL's THEOREM

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

where $X(k)$; $(0 \le k \le N-1)$ is a DFT of $x(n)$; $(0 \le n \le N-1)$

soln

$$L.H.S = \sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n)\, x^*(n)$$

From the synthesis eqn

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-nk} \quad \{\text{Defn of IDFT}\}$$

$$x^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X^*(k)\, W_N^{nk}$$

Substituting

$$\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n) \left[\frac{1}{N} \sum_{k=0}^{N-1} X^*(k)\, W_N^{nk}\right]$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) \left[\sum_{n=0}^{N-1} x(n)\, W_N^{nk}\right]$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X^*(k)\, X(k)$$

i.e. $$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

This expression is the Parseval's theorem. It expresses the energy in the finite duration signal $x(n)$ in terms of freq components $X(k)$.

RHS → mean square spectral amplitude

LHS → Sum of squared magnitude of the time series.

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* DFT of some useful signals.

i) $\delta(n)$ — impulse

$$DFT\{\delta(n)\} = \sum_{n=0}^{N-1} \delta(n) W_N^{kn}$$

$\because$ $\delta(n)$ is present only at $n=0$ & has a zero value for all other values of 'N'. The summation on the RHS is zero for any value of 'n' other than $n=0$

when $n=0$ $\quad RHS = \delta(0) W_N^{k0}$

$$= 1$$

ii) $\delta(n-n_0)$

$$DFT\{\delta(n-n_0)\} = \sum_{n=0}^{N-1} \delta(n-n_0) W_N^{kn}$$

$\delta(n-n_0)$ has a value 1 only when $n=n_0$ & has a zero value for any other value of n

$$\therefore RHS = \delta(n-n_0) W_N^{kn_0}$$
$$= \delta(0) W_N^{kn_0}$$
$$= 1 \cdot W_N^{kn_0}$$

iii) 1 (constant)

Let us first find

$$IDFT\{\delta(k)\} = \frac{1}{N} \sum_{k=0}^{N-1} \delta(k) W_N^{-kn}$$

$\delta(k)$ takes a value 1 only when $k=0$ & has value 0 for any value of k

$$\therefore RHS = \frac{1}{N} \delta(0) W_N^{-0 \cdot N}$$

$$= \frac{1}{N}$$

$$\frac{1}{N} \xleftrightarrow{DFT} \delta(k)$$

$$\therefore 1 \xleftrightarrow{D.FT} N\delta(k)$$

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iv) $x(n) = \cos n\omega_0 \quad ; \quad 0 \le n \le N-1$ (Sinusoid)

$= \cos\left(\frac{2\pi n k_0}{N}\right)$ (sinusoid)

$$\cos \frac{2\pi n k_0}{N} = \frac{e^{j\frac{2\pi n k_0}{N}} + e^{-j\frac{2\pi n k_0}{N}}}{2}$$

$$= \frac{1}{2} e^{j\frac{2\pi n k_0}{N}} + \frac{1}{2} e^{-j\frac{2\pi n k_0}{N}}$$

$$= \frac{1}{2} W_N^{-ln} + \frac{1}{2} W_N^{ln}$$

where $l = k_0$

$$DFT\left\{\cos \frac{2\pi n k_0}{N}\right\} = \frac{1}{2} DFT\left\{1 . W_N^{-ln}\right\} + \frac{1}{2} DFT\left\{1 . W_N^{ln}\right\}$$

we know

$$DFT\{1\} = N\,\delta(k)$$

$$\& \; x(n)\, W_N^{\pm ln} \xleftrightarrow{DFT} X(k \pm l) \quad \{\text{Freq translation}\}$$

$$\therefore DFT\left\{\cos \frac{2\pi n k_0}{N}\right\} = \frac{1}{2} N\,\delta(k-k_0) + \frac{1}{2} N\,\delta(k+k_0)$$

$$= \frac{N}{2}\,\delta(k-k_0) + \frac{N}{2}\,\delta(k+k_0-N)$$

Alternate Method

$$x(n) = \frac{1}{2} e^{jn\omega_0} + \frac{1}{2} e^{-jn\omega_0}$$

$$X(k) = \frac{1}{2}\sum_{n=0}^{N-1} e^{jn\omega_0} e^{-j\frac{2\pi kn}{N}} + \frac{1}{2}\sum_{n=0}^{N-1} e^{-jn\omega_0} e^{-j\frac{2\pi kn}{N}}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} e^{-jn\left(\frac{2\pi k}{N} - \omega_0\right)} + \frac{1}{2}\sum_{n=0}^{N-1} e^{-jn\left(\frac{2\pi k}{N} + \omega_0\right)}$$

$$\omega_0 = \frac{2\pi k_0}{N}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} e^{-jn\frac{2\pi}{N}(k-k_0)} + \frac{1}{2}\sum_{n=0}^{N-1} e^{-jn\frac{2\pi}{N}(k+k_0)}$$

The sum will turn out to be zero unless $k = k_0$ & $k = -k_0$

when $k = k_0$ & $k = -k_0$

$$\therefore X(k) = \begin{cases} \frac{N}{2} & ; \; k = k_0 \;\&\; k = -k_0 \\ 0 & ; \text{ otherwise} \end{cases}$$

$$= \frac{N}{2}\,\delta(k-k_0) + \frac{N}{2}\,\delta(k+k_0)$$

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* DFT of Sinusoids

$x(t) = A\cos(2\pi F_0 t + \phi)$ Find it's DFT

Soln

Let T_s be the sampling interval

$$x(nT) = A\cos(2\pi F_0 n T_s + \phi)$$

$$= A\cos\left(2\pi \frac{F_0}{F_s} n + \phi\right)$$

$$= A\cos(\omega_0 n + \phi)$$

$$= \frac{A}{2} e^{j(\omega_0 n + \phi)} + \frac{A}{2} e^{-j(\omega_0 n + \phi)}$$

$$= \frac{A}{2} e^{j2\pi \frac{F_0}{F_s} n} \cdot e^{j\phi} + \frac{A}{2} e^{-j2\pi \frac{F_0}{F_s} n} \cdot e^{-j\phi}$$

$$= \frac{A}{2} e^{j\phi} W_N^{-ln} + \frac{A}{2} e^{-j\phi} W_N^{ln}$$

$$DFT\{x(n)\} = \frac{A}{2} e^{j\phi} DFT\{1 \cdot W_N^{-ln}\} + \frac{A}{2} e^{-j\phi} DFT\{1 \cdot W_N^{ln}\}$$

$DFT\{1\} = N\delta(k)$ & $x(n) W_N^{-ln} \Longleftrightarrow X(k \mp l)$

$$= \frac{A}{2} e^{j\phi} N\delta\left(k - \frac{F_0}{F_s}\right) + \frac{A}{2} e^{-j\phi} N\delta\left(k + \frac{F_0}{F_s}\right)$$

$$= \frac{A}{2} e^{j\phi} N\delta\left(k - \frac{F_0}{F_s}\right) + \frac{A}{2} e^{-j\phi} N\delta\left(k + \frac{F_0}{F_s} - N\right)$$

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